M.Sc. Sem III 12th Paper

FLUID MECHANICS

HYDRODYNAMICS - FUNDAMENTALS

* velocity of the particle $= \dfrac{d\vec{r}}{dt} = \dot{\vec{r}}$

* consider any scalar point function $\phi(x,y,z,t)$ $= \phi(\vec{r},t)$ associated with a fluid in motion.

when the point (x,y,z) is fixed, the change in scalar point function ϕ in time δt

$$= \phi(x,y,z,t+\delta t) - \phi(\cancel{x,y},z,t)$$

$$= \phi(\vec{r}, t+\delta t) - \phi(\vec{r},t)$$

$\Rightarrow$ local time rate of change

$$= \frac{\partial \phi}{\partial t} = \lim_{\delta t \to 0} \frac{\phi(\vec{r}, t+\delta t) - \phi(\vec{r},t)}{\delta t}$$

* Similarly,

local time rate of change in vector point function $= \dfrac{\partial \vec{f}}{\partial t} = \lim\limits_{\delta t \to 0} \dfrac{f(\vec{r}, t+\delta t) - f(\vec{r},t)}{\delta t}$

* $u = \dfrac{dx}{dt} = \dot{x}, \quad v = \dfrac{dy}{dt} = \dot{y}, \quad \omega = \dfrac{dz}{dt} = \dot{z}$ — (A)

* $\vec{q} = u\vec{i} + v\vec{j} + \omega\vec{k}$

* $$\frac{d\phi}{dt} = \lim_{\delta t \to 0} \frac{\phi(\vec{r}+\delta\vec{r}, t+\delta t) - \phi(\vec{r}, t)}{\delta t}$$

But $\phi = \phi(x, y, z, t)$

$$\Rightarrow \frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial\phi}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial\phi}{\partial z}\cdot\frac{dz}{dt}$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial x}\cdot\dot{x} + \frac{\partial\phi}{\partial y}\dot{y} + \frac{\partial\phi}{\partial z}\dot{z}$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} + \omega\frac{\partial\phi}{\partial z} \quad \text{[using (A)]} \quad \text{— (1)}$$

Now, $(u\vec{i} + v\vec{j} + \omega\vec{k})\cdot\left(\vec{i}\dfrac{\partial\phi}{\partial x} + \vec{j}\dfrac{\partial\phi}{\partial y} + \vec{k}\dfrac{\partial\phi}{\partial z}\right)$

$$= u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} + \omega\frac{\partial\phi}{\partial z}$$

$$\Rightarrow (u\vec{i} + v\vec{j} + \omega\vec{k})\cdot\nabla\phi = u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} + \omega\frac{\partial\phi}{\partial z}$$

$$\Rightarrow \vec{q}\cdot\nabla\phi = u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} + \omega\frac{\partial\phi}{\partial z}$$

Putting this value in (1),

$$\Rightarrow \frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + (\vec{q}\cdot\nabla)\phi$$

Similarly, for vector point function,

$$\frac{d\vec{f}}{dt} = \frac{\partial\vec{f}}{\partial t} + (\vec{q}\cdot\nabla)\vec{f}.$$